**Optimal Resource Allocation for Multicast Flows in Wireless Networks**
L. Bui, R. Srikant, and A. Stolyar

**Model**
- Wireless network is modelled as a graph $G = (N, L)$.
- When node $n$ sends out a packet, all neighbours of $n$ can hear it, but only some of them choose to receive it, depending on whether that packet is forwarded to them or not.
- Both unicast and multicast traffic.
- Unicast flow: begin node, end node.
- Multicast session: directed multicast tree whose root is the source node.
- **Multi-rate** multicast: receivers in a multicast session can be served at different data rates.

**Network Utility Maximization**
- **Congestion Control problem:**
  $$\max \quad U_{1E}(x_{1E}) - p_{1E}x_{1E} + U_{1F}(x_{1F}) - p_{1F}x_{1F} + U_{2}(x_{2}) - p_{2A}x_{2}$$

- **Network Scheduling problem:**
  $$\max \quad \mu_{1C}(p_{1E} - p_{1C}) + \mu_{1D}(p_{1F} - p_{1D}) + \mu_{1B}(p_{1C} + p_{1D} - p_{1B}) + \mu_{1AP1B} + \mu_{2AB}(p_{2A} - p_{2B}) + \mu_{2BC}(p_{2B} - p_{2C}) + \mu_{2CEP2C}$$

  $$\text{s.t. } \mu \in \Gamma$$

**Solution: Shadow queues**
- A fictitious queueing network sending fictitious packets in the opposite direction.
- The departures from the fictitious queues serve as tokens for the generation of real packets.

**QoS control: Delays**
- Tokens inform the source of the amount of resources reserved for it.
- Source can use this information, but sends at a smaller rate to reduce delays.
- Can be also used for unicast traffic.

**Algorithms**
- Congestion control: generate tokens for multicast traffic and packets for unicast traffic.
- Scheduling: jointly schedule unicast real traffic and multicast shadow traffic (tokens).
- Multicast packet generation: generate as many packets as received tokens.