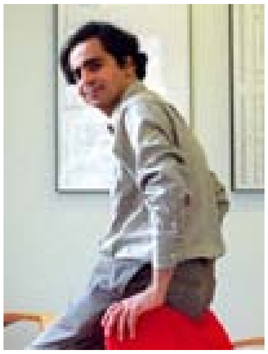
Illinois Center for Wireless Systems

ICWS Seminar Series



Eigen-distributions of Random Matrices From Binary Vector Spaces

Professor Vahid Tarokh
School of Engineering and Applied Sciences
Harvard University

Monday, October 4, 2010
Room 141 Coordinated Science Lab

Abstract: One of the most central statistical results of 20th century is the Marchenko-Pastur law which (among other things) states that singular values of (normalized) +-1 matrices with all i.i.d. elements asymptotically follow the Marchenko-Pastur distribution.

In this work, we consider a (normalized) +-1 random matrix A whose rows are formed by drawing from elements of a k dimensional binary vector space V \subseteq GF(2)^n (V is sometimes referred to as a k-dimensional binary linear block code of length n) uniformly and in an i.i.d. manner, and then applying the mapping 0--> 1 and 1 ---> -1.

We establish and prove (in our opinion) a very striking result: Assume that all nonzero vectors of V^{\perp}, the dual of the vector space V (also referred to as the dual code) have at least d non-zero elements (i.e. minimum Hamming distance of the dual code is >= d). We produce an almost sure bound on the distance between the distribution of singular values of A and the Marchenko-Pastur distribution. It follows from our bound that as d grows large, the eigen-behavior of A (formed from a structured code) is almost surely identical to the eigen-behavior of an i.i.d +-1 matrix!

Philosophically, our result implies that the randomness of a code is related to the minimum distance of the dual code. We briefly discuss the implications of this result on designing random sequences. We then seek after other generalizations of our results. Here, we produce no mathematical proofs. We discuss (empirically verified) generalizations of our result to the spectra of products of structured matrices, and to the similarity of eigenvalues of structured matrices to those given by the Girko circular rule, etc.). The proof of these generalizations may require development of sophisticated computational results in Voiculescu's free probability theory. If these generalizations are proved, then one can compute tight bounds on the spectral efficiency of "practical" (non-random) CDMA systems that use deterministic sequences (such as shift register sequences).

This is a joint work with Behtash Babadi.

Bio: Vahid Tarokh worked at AT&T Labs-Research and AT&T wireless services until August 2000, where he was the head of the Department of Wireless Communications and Signal Processing. In September 2000, he joined the Department of Electrical Engineering and Computer Sciences (EECS) at MIT as an associate professor. In June 2002, he joined Harvard University as a Gordon McKay Professor of Electrical Engineering . Since July 2005, he is a Hammond Vinton Hayes Senior Fellow of Electrical Engineering at Harvard University, and a Perkins professor.

Tarokh's output of the last 20 years is summarized in less than 50 research journal papers that have been cited more than 18000 times by other scholars. He is the recipient of a number of awards, and holds 2 honorary degrees.